

A NONEQUILIBRIUM VISCOUS SHOCK-WAVE LAYER
 IN THE VICINITY OF THE CRITICAL POINT
 WITH THE ASSOCIATED HEAT EXCHANGE TAKEN INTO ACCOUNT

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Results are presented in this article of numerical calculations of a viscous shock layer with the associated heat exchange in the vicinity of the critical point of a spherical blunt body taken into account in the presence of nonequilibrium chemical processes in the shock layer and on the surface of the body about which the flow takes place. A number of papers [1-4], in which specification of the surface temperature of the obstacle was utilized, have been devoted to the numerical investigation of a nonequilibrium viscous shock layer. At the same time the surface temperature of a body varies in actual flight due to heating, and together with this there is catalytic activity of the material, which appreciably complicates the problem and necessitates the simultaneous treatment of the course of processes in the gaseous and solid phases. The use of a separate formulation is difficult in this case, since the formulas for the thermal flux from the gaseous phase are of an estimative nature [5] when a volume nonequilibrium chemical reaction is present for a surface having an arbitrary catalytic activity. Taking account of the associated heat exchange has been done before for a number of problems of boundary-layer theory [6, 7], and in this case it has permitted determining the characteristics which are most important from the practical standpoint under conditions of flight along a specified trajectory, as well as under specified time-independent conditions of flight at altitudes at which the approximation of a viscous shock layer is valid. The effect of catalytic activity is discussed for a number of surface materials, and it is shown that the use of the formulas of boundary-layer theory can appreciably distort the behavior of the surface temperature as a function of time for a certain altitude range.

1. Assuming air to be a binary mixture of atoms and molecules, we write within the framework of the theory of a viscous shock layer a system of equations in the vicinity of a frontal critical point in the form

$$\frac{1}{\eta_s^2} \frac{d}{d\eta} \left(l \frac{d^2 f}{d\eta^2} \right) + f \frac{d^2 f}{d\eta^2} = \frac{1}{2} \left[\left(\frac{df}{d\eta} \right)^2 - 2 \frac{\rho_\infty}{\rho} N \right]; \quad (1.1)$$

$$\frac{1}{\eta_s^2} \frac{d}{d\eta} \left(\frac{l}{Pr} \frac{dH}{d\eta} \right) + f \frac{dH}{d\eta} + \frac{1}{\eta_s^2} \frac{d}{d\eta} \left[\frac{(Le-1)}{Pr} l h^0 \frac{dc}{d\eta} \right] = 0; \quad (1.2)$$

$$\frac{1}{\eta_s^2} \frac{d}{d\eta} \left(\frac{l}{Sc} \frac{dc}{d\eta} \right) + f \frac{dc}{d\eta} = - \frac{1}{2} \frac{1}{\beta} \frac{w}{\rho}; \quad (1.3)$$

$$p_s = \frac{\rho R (c + 1) T}{2M}. \quad (1.4)$$

In contrast to the generally adopted form of [1, 2], the variable $\eta = \frac{1}{\eta_s} \int_0^y \sqrt{\frac{2\beta}{\rho_s \mu_s}} \rho dy$, where $\eta_s = \sqrt{\frac{2\beta}{\rho_s \mu_s}} \int_0^{y_s} \rho dy$

is a coordinate which characterizes the position of the shock wave, was used in writing (1.1)-(1.3). Due to the hypersonic flow velocities around the body, it was assumed that $p = p_s \approx \rho_\infty V_\infty^2$ in the shock layer. The function N in (1.1), which characterizes the variation of the pressure gradient dP/dx across the shock layer, was taken in the form

$$N = 1 + \eta_s \sqrt{\frac{\rho_s}{\rho_\infty}} \frac{1}{\sqrt{2Re_s}} \int_0^1 \left(\frac{df}{d\eta} \right)^2 d\eta.$$

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The heat-conduction equation for the body around which the flow occurs is of the form

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial y_1} \left(\pi_1 \frac{\partial \Theta}{\partial y_1} \right). \quad (1.5)$$

Taking the Rankine-Hugoniot relationships into account on the shock wave, we write the boundary conditions as follows:

$$\eta = 1, \quad \frac{\partial f}{\partial \eta} = 1, \quad f = \frac{1}{\eta_s} \sqrt{\frac{\text{Re}_s \rho_\infty}{2 \rho_s}}, \quad c = c_\infty = 0, \quad H = 1. \quad (1.6)$$

Taking the arbitrary catalytic activity of the surface of the body into account, we have

$$\eta = 0, \quad \frac{\partial f}{\partial \eta} = 0, \quad f = 0, \quad \frac{l_w}{\text{Sc}_w} \frac{1}{\eta_s} \frac{dc}{d\eta}(0) = \zeta_w C_w, \quad (1.7)$$

$$\sqrt{2\text{Re}_s} \sqrt{\frac{\rho_s}{\rho_\infty}} \text{Pr}_w \frac{\mu_s}{\mu_w} \frac{\lambda_w}{\lambda_{1*}} \frac{H_s}{c_{p_w} T_0} \left[\frac{l_w}{\text{Pr}_w} \frac{1}{\eta_s} \frac{dH}{d\eta}(0) + \frac{l_w}{\text{Pr}_w} (\text{Le}_w - 1) h^0 \frac{1}{\eta_s} \frac{dc}{d\eta}(0) \right] - \pi_\sigma \Theta_w^k = -\pi_{1*} \frac{\partial \Theta}{\partial y_1}(0).$$

on the interface of the gaseous and solid phases.

We write the initial boundary condition for Θ in the form

$$\Theta(0, y_1) = \Theta(\tau, L/R_N) = 1. \quad (1.8)$$

The last of the boundary conditions (1.7) is characteristic for problems of associated heat exchange and expresses the law of energy conservation on the media interface. One should note that the calculated values of the surface temperature of the body are small for the section of the model trajectory being discussed and one can neglect the contribution of radiation from the material in the law of energy conservation (1.7). The relation between the total enthalpy, the temperature, and the concentration in the shock layer was used in the form [1]

$$H = \left(\frac{9}{2} + \frac{c}{2} \right) \frac{R}{2M} \frac{T}{H_s} + ch^0 \quad (1.9)$$

in the numerical integration.

The term from the source w/ρ is written in the usual way:

$$\frac{w}{\rho} = -4K_r \left(\frac{p}{RT} \right)^2 \left[\frac{c^2}{1-c} - \frac{K_e}{4p} (1-c) \right].$$

The notation

$$l = \rho_w / \rho_s \mu_s, \quad \text{Re}_s = \frac{\rho_\infty V_\infty R_N}{\mu_s}, \quad \beta = \frac{dU_s}{dx} \Big|_{x=0} = \frac{V_\infty}{R_N},$$

$$\Theta = \frac{T}{T_0}, \quad \pi_1 = \frac{\lambda_1}{\lambda_{1*}}, \quad y_1 = -\frac{y}{R_N}, \quad \tau = \frac{t}{t_*},$$

$$l_* = \frac{R_N^2 \rho_1 c_{p1}}{\lambda_{1*}}, \quad \zeta_w = \frac{K_w l_w}{\mu_w} \sqrt{\frac{\rho_s \mu_s}{2\beta}}, \quad \pi_\sigma = \frac{\varepsilon_w \sigma R T_0^3}{\lambda_{1*}}$$

was used in writing the boundary-value problem (1.1)-(1.9).

Here and above f is the dimensionless stream function, $\partial f / \partial \eta = U / U_s$ is the dimensionless velocity, H and h^0 are the dimensionless enthalpy and thermal effect of the chemical dissociation reaction, Pr , Sc , and Le are the Prandtl, Schmidt, and Lewis numbers, respectively, ρ and c_p are the density and specific heat, μ and λ are the viscosity and thermal conductivity coefficients of the mixture, respectively, Θ is the dimensionless temperature in the solid body, c is the concentration of atoms, K_r , K_w , and K_e are the recombination rate constants in the gas and on the surface of the body, and the equilibrium constant, respectively, R is the universal gas constant, R_N is the radius of the blunt end, and L is the wall thickness. The subscripts w , s , ∞ refer to quantities on the surface of the body, directly behind the shock wave, and in the undisturbed flow, respectively. The subscript "1" refers to the characteristics of the solid body. The subscript "0" refers to the temperature of the body at the initial time, and the subscript "*" is assigned to a number of characteristic quantities.

As follows from (1.1)-(1.8), the assumption of quasistationarity of the processes in the gaseous phase was used in connection with the formulation of the problem, in accordance with estimates of the relaxation times of processes in the gaseous and solid phases.

In the numerical integration Pr and Sc were assumed to be constant quantities, a formula from [8] was used for the viscosity coefficient, and the expressions for K_r and K_e were taken from [1]. The temperature

dependence for the recombination rate constant K_w on the surface of the body was taken in the form of a power law, which approximates the results of experimental investigations of the recombination efficiency [9, 10] for a number of materials. In the calculations the thermal conductivity coefficient in the body λ_1 was assumed to be constant, and by virtue of this $\pi_1=1$; the thermophysical characteristics of different materials were taken from [11].

2. The numerical integration of the boundary-value problem (1.1)-(1.8) was performed with the help of a difference scheme obtained on the basis of an iterative interpolation method [12]. The error of the approximation of the original system of differential equations and boundary conditions was $O(\Delta \eta)^2$ and $O(\Delta y_1)^2 + O(\Delta \tau)$. The difference scheme employed permitted performing a stable calculation over a wide range of the initial parameters of the problem. With constant conditions in the oncoming flow a number of calculations were performed, prior to the establishment of steady conditions for occurrence of the process in the solid body, for which a comparison was made of the numerical and analytic solutions. The results of [1] were used as test checks.

Due to the assumption adopted concerning the quasistationarity of the processes in the gaseous phase, the computational procedure was constructed as follows: The fields of the characteristics in the gaseous phase were calculated for the known surface temperature at the initial time. When the necessary accuracy was attained for the desired distributions and η_s , the gradient of the enthalpy and concentration of the media interface was determined; then the temperature field in the solid phase and the surface temperature were determined from Eqs. (1.5) with the boundary and initial conditions (1.7) and (1.8) taken into account. The process was repeated in the way indicated above for a new value of the surface temperature; the new value of the altitude, the flow parameters at infinity, and the determining factors of the problem associated with them were determined under flight conditions along the trajectory.

In order that the calculation be stable at the altitudes at which the conditions for a chemical reaction to proceed are close to equilibrium ones, methods similar to those of [3, 13] were used, which permitted checking flight regions with rather large Re_s numbers in a number of cases.

The step $\Delta \tau$ was varied during the performance of the numerical calculations, and the step $\Delta \eta$ was varied in a number of alternatives.

As the calculations have shown, the surface temperature and thermal flux for constant parameters of the incoming flow corresponding to an altitude of 70 km differ strongly at the identical times, depending on the catalytic activity of the body. With velocity $V_\infty = 7.9$ km/sec, $T_0 = 300^\circ\text{K}$, and $R_N = 0.3$ m the calculations performed prior to establishment of steady conditions for occurrence of the process in the solid body showed that the steady value of Θ_w obtained for an ideal catalytic surface exceeds by almost one-and-one-half times the steady value of Θ found for $\xi_w = 0$. It was assumed in the numerical integration that $Pr = 0.72$, $Le = 1$, and $L/R = 0.5$.

It is interesting to note that the thermal flux curve found for the conditions indicated above for flow around a nickel surface is of a nonmonotonic nature, in contrast to the monotonic behavior of the thermal flux $q_w(\tau)$ for the limiting cases of catalytic activity. This situation is associated with the presence of two contradictory tendencies; on the one hand, as Θ_w increases, the convective part of the thermal flux decreases, and on the other hand, the diffusive part q_w , which is expressed in terms of the rate of a heterogeneous recombination reaction for which $\partial \xi_w / \partial \Theta_w > 0$, increases.

Actually, for a given model of air one can write

$$\frac{\mu_w}{Pr_w} \frac{\partial H}{\partial y}(0) = \frac{\lambda_w}{H_s} \frac{\partial T}{\partial y}(0) + \rho_w K_w h^0 c_w.$$

Since the rate of the heterogeneous reaction increases as the wall temperature goes up, the concentration of atoms on the surface of the body decreases, which leads to a further decrease of $q_w(\tau)$. But the presence of such an effect, which will depend in the general case on the range of variation of Θ_w , was first pointed out in [8].

The results of the solution of the associated problem upon a descent of the body along a model vertical trajectory with constant velocity, which are presented in Figs. 1-5, are of interest. The initial flight altitude is 90 km in this case, $V_\infty = 7.9$ km/sec, and $T_0 = 300^\circ\text{K}$. In doing these calculations the values of the radius of the blunt end $R_N = 0.06, 0.3, \text{ and } 1.5$ m were used for bodies made of tungsten and Pyrex, and the calculated values of Pr , Le , and L/R_N are indicated above. The discussion was conducted on the basis of the concept of a compressed viscous shock layer; the results are of a model nature in the altitude region near 90 km for a

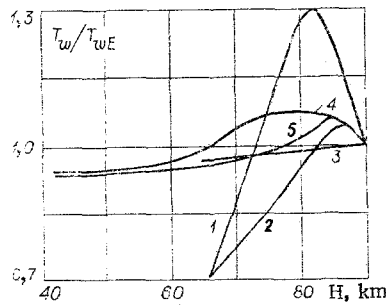


Fig. 1

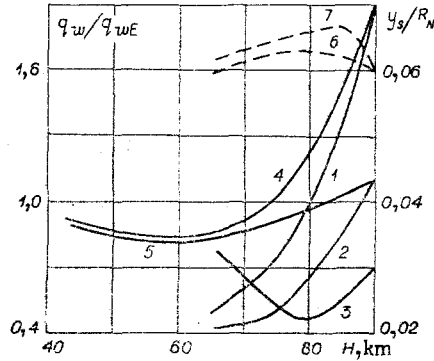


Fig. 2

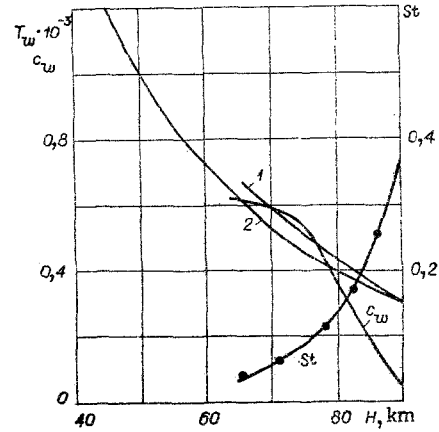


Fig. 3

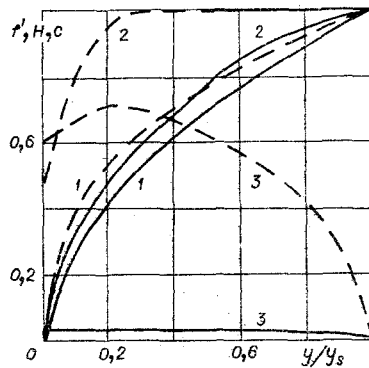


Fig. 4

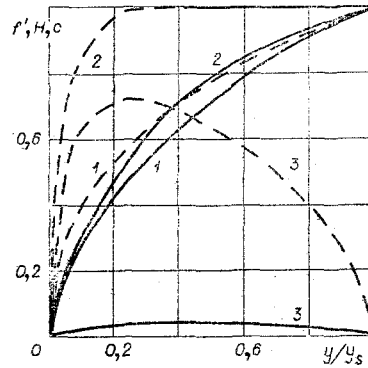


Fig. 5

blunt-end radius $R_N=0.06$ m, since for this version the values of Re_S at the indicated altitudes are rather small, and it is necessary to take account of the effects of a diffuse shock wave.

The ratios of the surface temperature and thermal flux q_w to the surface temperature T_{wE} and thermal flux q_{wE} found from the solution of the boundary-value problem (1.5) and (1.8), using the formulas of [14] for the thermal flux on the part of the gaseous phase, are given in Figs. 1 and 2 as a function of the flight altitude. Calculation of the time-dependent thermal conductivity equation with a specified thermal flux taken into account in connection with the equilibrium occurrence of a chemical reaction in the boundary layer permitted estimating the degree of applicability of the approximate formulas of boundary-layer theory for different materials of the body under the flight conditions indicated above.

Curves 1-3 of Figs. 1 and 2 are found for a body made out of Pyrex with the values $R_N=0.06, 0.3,$ and 1.5 m, respectively, and the curves 4 and 5 are obtained for a tungsten surface with $R_N=0.06$ and 0.3 m, respectively.

As is evident from Figs. 1 and 2, the use of formulas from the boundary-layer theory for the body made out of Pyrex, which has a low catalytic activity, may introduce an appreciable error in the determination of the quantities T_w and q_w ; this error increases as the blunt-end radius decreases. For large blunt-end radii the

effect of vorticity of the external flow weakens, and the thermal flux tends towards its own equilibrium value as the body descends (curve 3, Fig. 2); a thin boundary layer is formed simultaneously near the body, as the calculations show. The surface temperature in this case (curve 3, Fig. 1) agrees with T_{WE} within 4% limits. The nonmonotonic nature of the behavior of T_W/T_{WE} is caused, as follows from Fig. 1, by a variation of the ratio $q_W/q_{WE}(H)$, since at higher altitudes the thermal flux calculated from the theory of a compressed viscous shock layer can exceed by several times the thermal flux found from the theory of an asymptotically thin equilibrium boundary layer. The quantity q_W becomes less than the corresponding value q_{WE} upon a decrease in the flight altitude or an increase in R_N ; a minimum value of the ratio q_W/q_{WE} is realized for some altitude. Upon a further decrease in the altitude the recombination process in the viscous layer, which is accompanied by liberation of energy, increases, and the value of the thermal flux q_W increases.

As is evident from Figs. 1 and 2, the calculations for a coating made out of Pyrex were performed for altitudes no lower than 65 km, since the surface temperature in this case can reach temperatures at which damage occurs, and it is necessary to replace the original mathematical model.

The principal regularities of the behavior of T_W/T_{WE} and q_W/q_{WE} noted above are also characteristic for a body made out of tungsten, which has a high catalytic activity of its surface in comparison with Pyrex and is a good heat conductor. However, as one should expect, the minimum values of the q_W/q_{WE} curves in this case lie significantly above the corresponding values for Pyrex, and agreement of the temperatures T_W and T_{WE} is observed to an accuracy of about 8% over a wide range of flight altitudes.

Curves 6 and 7 in Fig. 2 correspond to a dimensionless withdrawal of the shock wave y_s/R_N as a function of the flight altitude for $R_N=0.3$ m, curve 6 is obtained for a coating made out of Pyrex, and curve 7 is for tungsten.

The dependences of T_W , c_W , and the Stanton number $St = q_W/\rho_\infty V_\infty (H_\infty - H_W)$ (H is the dimensional enthalpy) on flight altitude are presented in Fig. 3 for a number of alternatives. Curve 1 for T_W and the curves for c_W and St are obtained for Pyrex; $R_N=0.3$ m. Curve 2 corresponds to a surface temperature for $R_N=0.06$ m, and the coating is tungsten. The calculation for Pyrex is performed in this case to the altitude at which the surface temperature attained the temperature for damage to the material. As is evident from Fig. 3, the surface temperature of a body made out of Pyrex increases more rapidly with time than does the surface temperature of tungsten due to an appreciable difference in the thermal conductivity coefficients of the materials, notwithstanding the fact that in the latter case R_N is five times less and K_W for tungsten significantly exceeds the corresponding value of K_W for Pyrex. We also note that the surface temperature $T_W(H)$ (curve 2) during flight does not exceed $1.2 \cdot 10^3$ °K. The very low values found for T_W are caused by the short flight times (about 5 sec) for the specified model trajectory and the condition (1.8) adopted at $y_1 = h/R$.

As the calculations have shown, good agreement of the St numbers for similar R_N is observed in this case for bodies with different catalytic properties. (The values of the St numbers for $R_N = 0.3$ m are denoted by filled circles in Fig. 3, and the material is tungsten.) The indicated circumstance permits estimating the values of the thermal fluxes for coating materials in connection with the flight of bodies along a trajectory.

The fields of the dimensionless velocity $\partial/\partial\eta$ (curves 1), the dimensionless enthalpy H (curves 2), and the concentration c (curves 3) in the viscous shock layer at different times are presented in Fig. 4 (material is Pyrex) and Fig. 5 (material is tungsten) for $R_N=0.3$ m. The solid curves of Figs. 4 and 5 correspond to the initial time $t=0$, and the dashed curves correspond to $t=3$ sec. It is evident that the degree of dissociation in the shock layer is insignificant at the initial time. A decrease in the flight altitude leads to a clearer separation of the flow region into a boundary layer and a nonviscous part; the concentration of atoms in the shock layer increases, and the recombination process occurs near the surface, as is evident from Figs. 4 and 5; this process occurs very intensively on the tungsten surface and far more weakly on the coating made out of Pyrex.

Thus the investigation carried out with associated heat exchange taken into account shows the importance of a complex approach in connection with finding the determining characteristics T_W and q_W under flight conditions of bodies along specified trajectories.

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